Mutual Insurance Networks and Unequal Resource Sharing in Communities

Pascal Billand

Christophe Bravard

Sudipta Sarangi

Motivation

- Household income in developing countries varies but consumption is smooth
- Such economies usually lack formal insurance markets
- Informal arrangements enable communities in these countries to counter the effects of income variation
- We use a network to model these bilateral mutual insurance agreement

Some stylized facts

- Numerous studies (Townsend , 1994) have shown that informal insurance is not a community level phenomenon.
- In times of need individuals seek help not from the entire village but only from friends and family (Fafchamps and Lund, 2003)
- Townsend (1994) also shows that the sharing of resources is not equal

Objective

• Use these stylized facts to build a networks model

• Identify the structure of stable and efficient networks

 Examine when asymmetric /symmetric architectures can be stable among *ex ante* symmetric agents

• Examine the impact of agent heterogeneity

Model features

- Agents randomly obtain resources of two values: *high* or *low* (static setting)
- Pairs of agents insure each other by "sharing" resources (*hence risk*) to deal with these fluctuations
- Agents only insure those with whom they have a direct connection (immediate neighborhood)
- High endowment agents give low endowment agents in their immediate neighborhood a fixed amount

Model features

- Low endowment agents receive a fixed amount from each high endowment agent in their neighborhood
- When two agents insure each other, each of them increases her chance of obtaining a better payoff when she receives low resources and vice versa.
- High endowment agents always obtain higher benefits than those with low endowments (*no equal sharing norm*)

Model features

- Links are costly and costs depend on the number of links established by the agents:
 - For a link between agent *i* and *j*, the cost of a link increases the more links *i* has (*Benchmark Model*)
 - For a link between agent *i* and *j*, the cost of a link increases the more links *j* has (*Model with Heterogeneity*)

• Benefits: Each additional link is less valuable than the previous one (*strictly concave*)

Literature

- Townsend (1994)
- Bloch, Genicot and Ray (2008)
- Bramoulle and Kranton (2006, 2007)

Literature

- There exist pairwise stable networks in which individuals are in asymmetric positions relative to the risk they support.
 - In such networks agents who obtain the smallest amount of insurance are always linked together.
 - Those who obtain the highest/optimal amount of insurance: *Depends*.
- In efficient networks agents always obtain similar amounts of insurance
- Conflict between efficiency and stability

Model setup

- Players: N = {1,2, ..., n}, n ≥ 3, community of ex ante identical agents.
- Endowments: realizations are independent and identical across agents.

State 1: $\Theta > 0$ with probability p
State 0: 0 with probability 1-p

• Links of player *i*: $(g_{i,1}, ..., g_{i,i-1}, g_{i,i+1}, ..., g_{i,n})$ $-g_{i,j} = 1 \implies$ there exists a link between players *i* and *j* $-g_{i,j} = 0 \implies$ there does not exist a link between *i* and *j*

Model setup

- $g + g_{i,i}$: Adding a link
- g g_{i,j} : Deleting a link
- A network g is a *n* × *n* matrix where
 - $-g_{ij} = 1$ implies *i* and *j* have a risk sharing agreement
 - $-g_{ij} = 0$ implies lack of such an agreement
 - by convention $g_{ii} = 0$
 - Risk sharing links are mutual: $g_{ij} = g_{ji}$

• Chain

 $4 \rightarrow 1 \rightarrow 3$





- Chain
- $4 \rightarrow 1 \rightarrow 3$
- Connected
- Component





Chain

 $4 \rightarrow 1 \rightarrow 3$

- Connected
- Component
- Empty and Complete
- Star





- k-regular
- k₊-regular
- k_-regular
- Almost k regular





• $g_{|N'}$: subnetwork of g where $i \in N'$ and $j \in N'$ are linked iff they are linked in g.

• Neighbors: those directly linked to one another

- Agent *i*'s neighbors: $N_i(g) = \{j \in \mathbb{N}: g_{ij} = 1\}$.

- Agent *i*'s degree: $n_i(g) = |N_i(g)|$.

Concavity and Convexity

• For each function f let $\Delta f(x) = f(x) - f(x-1)$.

- For all *x*, *f* is
 - Concave: $\Delta f(x+1) \Delta f(x) \leq 0$
 - Convex: $\Delta f(x+1) \Delta f(x) \ge 0$

Transfers

• When agent *i* draws State 1 she *gives* $\delta \in (0,1)$ to each of her neighbors who draws State 0.

• When *i* draws State 0 she *receives* $\delta \in (0,1)$ from each of her neighbors who draw State 1.

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 \Rightarrow Agents are ex ante identical but benefits depend on the network architecture

Expected payoffs

 The expected payoff function of each agent has two parts:

 A benefits part involving uncertainty depending on the endowment realization

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 A benefits part involving uncertainty depending on the endowment realization

 A cost part which involves no uncertainty where costs depend on the number of links formed

Expected benefits

- CARA utility: $u_i(x) = 1 e^{-\rho x}$ where x is income and $\rho > 0$.
- Suppose agent *i* draws 0 and *k* of her neighbors draw state 1, then her benefits are: $b^{g}(0, k) = u_{i}(k\delta) = 1 - e^{-\rho k\delta}$

Expected benefits

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- Suppose agent *i* draws 1 and *k* of her neighbors draw state 1, then her benefits are: $b^{g}(1,k) = u_{i}(\Theta - (n_{i}(g) - k)\delta)$ $= 1 - e^{-\rho(\Theta - (n_{i}(g) - k)\delta)}$

Expected benefits

$$B_{i}(g) = \phi(n_{i}(g)) = p \sum_{k=0}^{n_{i}(g)} {n_{i}(g) \choose k} p^{k} (1-p)^{n_{i}(g)-k} b^{g}(1,k)$$
$$+ (1-p) \sum_{k=0}^{n_{i}(g)} {n_{i}(g) \choose k} p^{k} (1-p)^{n_{i}(g)-k} b^{g}(0,k),$$

Costs & Payoffs

 Player i's links costs depend only on the number of links formed by her: C_i(g) = f₁(n_i(g)) where f₁(·) is strictly increasing and convex.

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- Player i's links costs depend only on the number of links formed by her: C_i(g) = f₁(n_i(g)) where f₁(·) is strictly increasing and convex.
- Expected payoffs:

$$U_i(g) = B_i(g) - C_i(g)$$
$$= \Phi(n_i(g)) = \phi(n_i(g)) - f_1(n_i(g))$$

Definitions: Stability and efficiency

• A network is said to be pairwise stable if

- for all
$$g_{ij} = 1$$
, $U_i(g) \ge U_i(g - g_{ij})$ and $U_j(g) \ge U_j(g - g_{ij})$
and

- for all $g_{ij} = 0$, if $U_i(g) > U_i(g + g_{ij})$ then $U_j(g) < U_j(g + g_{ij})$

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• An efficient network $(W(g^e))$ is one that maximizes the sum of the expected payoffs of all the agents.

Properties of the Expected Neighborhood (ENB) Function

- **Proposition 1**: The ENB function of agent *i* is strictly increasing and strictly concave in the number of links she has formed.
 - Each agent prefers more insurance to less when costs of links are sufficiently low.
 - The marginal ENB of an i agent decreases with each additional link.

Properties of the Expected Neighborhood (ENB) Function

- Proposition 2: The marginal ENB function of agent i increases with Θ.
 - If the agent draws the State 0, then her ENB is not affected by her income.
 - Suppose the agent draws State 1. Then due to the concavity of the utility function, when her income increases, her marginal utility is affected less by the fact that she has to help one of her neighbors.

Existence

• Follows from a theorem of Erdös and Gallai (1960)

A finite sequence s = (d₁, d₂, ..., d_n) of nonnegative integers is <u>graphical</u>
 if there exists a network g whose nodes have
 degrees d₁, d₂, ..., d_n.

Existence

• A sequence $s = (d_1, d_2, ..., d_n)$ of nonnegative integers such that $d_1 \ge d_2 \ge \cdots \ge d_n$ and whose sum is even is graphical if and only if $\sum_{i=1}^r d_i \le r(r-1) + \sum_{i=r+1}^n \min\{d_i, r\}$ for every $r, 1 \le r < n$

-Erdös and Gallai (1960)

Existence

Lemma 1: Let *n* and *k* be non-negative integers with *n* > *k*.

1. Let *n* or *k* be even. Then the sequence *s* = (*k*, ..., *k*) is graphical.

2. Let n and k be odd. Then the sequences s = (k+1, ..., k), s' = (k-1, ..., k) are graphical.

Lemma 1: Proof Sketch

1. Let *n* or *k* be even. Then the sequence s = (k, ..., k) is graphical.

Sketch: Let n > k > 0. Since n or k is even, the sum of s is even. So we have,

 $rk \le r(r-1) + \sum_{i=r+1}^{n} \min\{k,r\}$ for every $r, 1 \le r < n$

Case 1. $r \le k \Longrightarrow k \le n - 1$. Always true.

Lemma 1: Proof Sketch

Let *n* or *k* be even. Then the sequence s = (k, ..., k) is graphical.

Case 2. r > k. Note that since r < n and r > k. What happens when k = n-1? Complete network. When k = 0, empty network. In these two polar cases, the *s* is graphical.

So we have to consider 0 < k < n-1. Simple manipulation shows that this holds and Case 2 is satisfied.

Pairwise Stable Networks

- Φ is strictly concave \Rightarrow there exists \hat{k} such that $\Phi(\hat{k}) > \Phi(k)$ for all $k \neq \hat{k}$.
- Let $\mathcal{M}(g) = \{i \in N : n_i(g) \neq \hat{k}\}.$
- Proposition 3: Network g is pairwise stable iff for every agent i ∈ M(g), n_i(g) < k̂ and g_{|M(g)} is complete. Moreover, (a) if n or k̂ are even, then k-regular networks are always pairwise stable, and (b) if n or k̂ are odd the, k̂_-regular networks are always pairwise stable.
Pairwise Stable Networks: Intuition

- Suppose g is pairwise stable.
 - No agent will for more than \hat{k} links since it is the optimal.
 - If i and j are not linked and have less than \hat{k} links, they will form the link. Hence for $i \in \mathcal{M}(g)$, $n_i(g) < \hat{k}$ and $g_{|\mathcal{M}(g)}$ is complete.
- Suppose $i \in \mathcal{M}(g)$, $n_i(g) < \hat{k}$ and $g_{|\mathcal{M}(g)}$ is complete.
 - Since $g_{|\mathcal{M}(g)}$ is complete, no more links can be formed.
 - Since $n_i(g) < \hat{k}$, no agent will delete a link.

Pairwise Stable Networks: Intuition

- Suppose n or \hat{k} is even.
 - By Lemma 1 the sequence $s = (\hat{k}, \hat{k}, ..., \hat{k})$ is graphical.
 - So there exists a \hat{k} -regular network that is pairwise stable.
- Suppose n and \hat{k} are odd.
 - Again by Lemma 1, we can show that there exists a \hat{k}_{-} -regular networks that is pairwise stable.

Pairwise stable networks in pure strategies always exist for our payoff function. (Lemma 1 + Proposition 3)

Pairwise Stable Networks

• Let $\hat{k} = 3$ and $\mathcal{M}(g) = \{1,2,3\}$. Then the network shown here satisfies *Proposition 3*.



Pairwise Stable Networks

• Let $\hat{k} = 3$ and $\mathcal{M}(g) = \{1,2,3\}$. Then the network shown here satisfies *Proposition 3*.



- Note that since players are ex ante identical and only the degree matters, the identity of those in $\mathcal{M}(g)$ cannot be fixed.
- If links costs are zero, then the complete network is pairwise stable.

Efficient Networks

Proposition 4: Suppose n or \hat{k} are even. Then \hat{k} regular networks are the unique efficient networks. Suppose n and \hat{k} are odd. If $\Phi(\hat{k} + 1) < \Phi(\hat{k} - 1)$, then k_- -regular are the unique efficient networks. If $\Phi(\hat{k} + 1) > \Phi(\hat{k} - 1)$, then k_+ -regular are the unique efficient networks.

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 Examination of Propositions 3 and 4 shows that there exist pairwise stable networks that are not efficient.

Efficient Networks: Intuition

Suppose n or \hat{k} are even. Then \hat{k} -regular networks exists by Lemma 1. They also maximize each agents payoffs and are efficient.

Suppose n and \hat{k} are odd. Then by Lemma 1, \hat{k}_+ -regular and \hat{k}_- -regular networks exist. Say all agents except i can form \hat{k} links and are maximizing their payoffs. Since i cannot for \hat{k} links and Φ is concave, when $\Phi(\hat{k} + 1) < \Phi(\hat{k} - 1)$, she forms k-1 links. When $\Phi(\hat{k} + 1) > \Phi(\hat{k} - 1)$, then she forms k+1 links.

Income Heterogeneity

- Each link costs F > 0.
- Rich agents: N^{Θ} and poorer agents: $N^{\Theta'}$
- In State 0 everyone gets 0, but in State 1, $\Theta > \Theta'$.

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- By Proposition 1, \hat{k}_{Θ} and $\hat{k}_{\Theta'}$ exist.
- By Proposition 2, $\hat{k}_{\Theta} \geq \hat{k}_{\Theta'}$.
- Let $\mathcal{M}'(g)$ be the set of agents who have not formed either \hat{k}_{Θ} or $\hat{k}_{\Theta'}$ links.
- Let $|N^{\Theta}|$ and $|N^{\Theta'}|$ be even.

Income Heterogeneity: Pairwise stable networks

Proposition 5: Network g is pairwise stable if and only if for every agent $i \in \mathcal{M}'(g)$, $n_i(g) < \hat{k}_x$, $x \in \{\Theta, \Theta'\}$ and $g_{|\mathcal{M}'(g)}$ is complete.

The idea is very similar to that of Proposition 3 and the proof is similar.

Income Heterogeneity: Pairwise stable networks

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The idea is very similar to that of Proposition 3 and the proof is similar.

Corollary 2: There exists F such that only agents in N^{Θ} form links.

Income Heterogeneity: Pairwise stable networks

 $\hat{k}_{\Theta} \geq \hat{k}_{\Theta'} \text{ does not imply that for } i \in N^{\Theta} \text{ and } i' \in N^{\Theta'}$ we will have $n_i(g) \geq n_{i'}(g)$.

Let $N^{\Theta} = \{1,2\}$ and $N^{\Theta'} = \{3,4\}$. Let $\hat{k}_{\Theta} = 2$ and $\hat{k}_{\Theta'} = 1$.



Generous and miserly agents

- *N* consists of two types of agents:
- Generous: When agent *i* draws State 1 she gives $\delta^G \in (0,1)$ to each of her neighbors who draws State 0.
- Miserly: When agent *i* draws State 1 she gives $\delta^M \in (0,1)$ to each of her neighbors who draws State 0.
- $\delta^G > \delta^M$
- Each link has a fixed cost *F* > 0.

Results

- Marginal benefit of a *GG*-pairing > Marginal benefit of a *GM*-pairing.
- Marginal benefit of a *MG*-pairing > Marginal benefit of a *MM*-pairing.

Results

- Marginal benefit of a GG-pairing > Marginal benefit of a GM-pairing.
- Marginal benefit of a *MG*-pairing > Marginal benefit of a *MM*-pairing.
- There exists an *F* for which in a stable network all *G*-agents and *M*-agents form *2 separate complete networks* with no links between the 2 groups.
- For sufficiently small *F, G*-agents will also link to *M*-agents in a stable network.

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- They require costly time and effort.
- Costs of links increase as an agent forms more links.

- One of the biggest limitations of informal insurance is that it is "informal."
- They require costly time and effort.
- Costs of links increase as an agent forms more links.
- What if they also depend on the number of links of the partner?

$$C_{i}(g) = f_{1}n_{i}(g) + \sum_{l \in N_{i}(g)} f_{2}(n_{l}(g))$$

where $f_2(.)$ is strictly increasing and convex.

• $C_i(g+ij) - C_i(g) = \Delta f_1 n_i(g+1) + f_2(n_j(g+1))$

 \Rightarrow This difference is strictly positive given the properties of f_1 and f_2 .

• The benefits function is unchanged.

 $\Delta U_i(g, g_{ij}) = [B_i(g + g_{ij}) - C_i(g + g_{ij})] - [B_i(g) - C_i(g)]$

 $= \gamma(n_i(g)+1, n_j(g)+1)$

$$\begin{split} \gamma(n_i(g) + 1, n_j(g) + 1) &= \Delta \phi(n_i(g) + 1) - \Delta f_1 n_i(g + 1) \\ &- f_2(n_j(g + 1)) \end{split}$$

$$\gamma(n_i(g)+1, n_j(g)+1) = \Delta \phi(n_i(g)+1) - \Delta f_1 n_i(g+1) - f_2(n_j(g+1))$$

• γ is strictly decreasing in its first argument since $\Delta \phi(.)$ is strictly decreasing by Proposition 1 and $\Delta f_1(.)$ is strictly increasing.

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- γ is strictly decreasing in its second argument since $f_2(.)$ is strictly increasing.

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- γ is strictly decreasing in its first argument since $\Delta \phi(.)$ is strictly decreasing by Proposition 1 and $\Delta f_1(.)$ is strictly increasing.
- γ is strictly decreasing in its second argument since $f_2(.)$ is strictly increasing.

 \Rightarrow The marginal expected payoff function of agent i is decreasing in both arguments.

- There exists *k** such that
 - $-\gamma(k^*, k^*) \ge 0$ and
 - there is no $k, k > k^*$ such that $\gamma(k,k) \ge 0$. $\Rightarrow \gamma(k, k) > 0$ for all $k < k^*$.
- $S_l(g)$: set of agents who have *l* links in *g*.

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 - $\Rightarrow \gamma(k, k) > 0$ for all $k < k^*$.
- $S_l(g)$: set of agents who have *l* links in *g*.

Proposition 7: A pairwise stable network always exists.

Pairwise stable networks

• If g* is a pairwise stable networks then the following conditions are satisfied:

Q1: If $l, l' > k^*$, then there is no link between an agent who belongs to S_l and an agent who belongs to $S_{l'}$ in g^* . If $l, l' < k^*$, then there is a link between an agent who belongs to S_l and an agent who belongs to S_l and an agent who belongs to $S_{l'}$ in g^* .

Q2: Suppose $l' \le l < k^*$ and $k^* < k' \le k$. If there is a link between an agent who belongs to S_l and an agent who belongs to S_k in g^* , then there is a link between an agent who belongs to $S_{l'}$ and an agent who belongs to $S_{k'}$ in g^* .

Q3: If $\gamma(k^*+1, k^*) < 0$, then there is no agent with more than k^* links.

Pairwise stable networks

- g is pairwise stable network with $k^* = 4$, $S_5 = \{1,2\}, S_4 = \{3,4,5\}$ and $S_3 = \{6,7\}.$
- g satisfies Q1.



Pairwise stable networks

- Suppose *n* or *k** are even, then there exists a *k*-regular network which is pairwise stable.
- Suppose n and k* are odd, then there exists an almost k-regular network which is pairwise stable.







In a pairwise stable network...

- Ex ante identical agents can end up with different amounts of insurance.
- Agents with the highest levels of insurance or "insurance leaders" (more than k* links) are not connected to each other,
- Agents with the lowest levels of insurance <u>are</u> connected to each other (solidarity affect).
- Those with k* links can be connected to those with more or less links.

In a pairwise stable network...

- Even in regular networks, agents may not receive the same amount of insurance
- Moreover, it is possible to find the difference between the maximal and minimal amounts of insurance for any stable network
- Ex ante identical agents can end up with different amounts of insurance like in a *star* network

Efficient networks

• Let g^e be an efficient network. Then g^e is either a *k*-regular or almost *k*-regular network.

Intuition: The welfare function W(g) is the sum of payoffs of all agents. Hence it is concave. It follows that there exits a k^e number of links, $k^e \in \{0, 1, ..., n - 1\}$ such that W(g) is maximal.

Efficient networks

- Hence when *n* or *k* is even only *k*^{*}-regular networks can be stable and efficient.
- Let n be even. Then in the k*-regular pairwise stable network, agents will form <u>at least</u> the same number of links as in the efficient network.

⇒ Thus efficient and pairwise stable networks do not always coincide. Stable networks can be overconnected!

- We show that agents can always form stable mutual insurance agreements on their own
- Both asymmetric and symmetric (*regular* and *almost regular* networks) insurance agreements can arise in stable networks.
- We find that those with less than the optimal links will form a complete network.
- Cannot pin down what type of networks will be formed by those with the optimal number of links.

- With identical agents we cannot say more about the identity of the players in these groups.
- In efficient networks all agents have similar amounts (*regular* and *almost regular* networks) of insurance.
- Efficient and Stable networks may to coincide. In fact under cost heterogeneity, stable networks may be overconnected.

- With rich an poorer agents, we can have situations where the rich link to each other and the poor have no insurance.
- A richer person may have fewer links than a poorer person and may even have no links.
- With heterogeneity in the giving parameter we find that for stable networks there exist parameters under which the GG-pairings and MM-pairings form two separate complete networks

- When costs of link formation depend both the number of links each player has results are similar.
- But it is now possible to have insurance leaders who have more than k* and never connect to each other.
 Those with the lowest levels of insurance do connect to each other.
- A wide variety of insurance agreements are possible even ex ante among identical agents.
Thank you